# Section 10.1
## Parametric Functions

Instead of relating y directly to x, we will now define x and y as functions of a parameter t!!!

$$x = f(t)$$
$$y = g(t)$$

## Example

A ball is thrown straight up with initial velocity...

y = height (t)

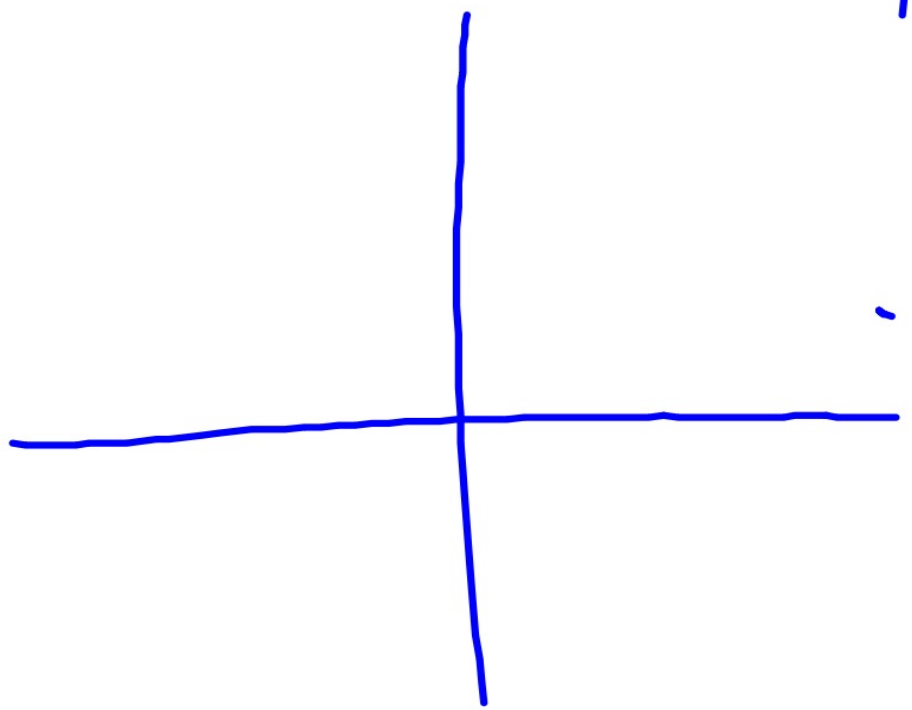
x = horizontal (t)

## Domain and Range

$x = t$
$y = t^2$

$y = x^2$

$t = 0 = (0, 0)$
$t = (-1) = (-1, 1)$



**Sketch the parametric curves and identify those which define y as a function of x. In each case, eliminate the parameter to find an equation that relates x and y directly.**

a.) $x = \cos t$ and $y = \sin t$ for $t$ in the interval $[0,2\pi)$

b.) $x = 3 \cos t$ and $y = 2 \sin t$ for $t$ in the interval $[0,4\pi]$

c.) $x = \sqrt{t}$ and $y = t - 2$ for $t$ in the interval $[0,4]$.

Given $x = \cos t$, $y = \sin t$, Eliminate the parameter…

**Given $x = 2t + 4$ and $y = 3t^2 - t$, eliminate the parameter**

**Given $x = 2t - 4$ and $y = t^2 + 3t$. Eliminate the paramter.**

## Parametric Differentiation Formulas

If x and y are both differentiable functions of t and if $dx/dt \neq 0$, then

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

If $y' = dy/dx$ is also differentiable function of t, then

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(y') = \frac{dy'/dt}{dx/dt}$$

The slope of the curve is still $dy/dx$

The concavity still depends on $d^2y/dx^2$

Consider the curve defined parametrically by $x = t^2 - 5$ and $y = 2 \sin t$ for $0 \leq t \leq \pi$

a.) Sketch a graph of the curve in the viewing window [-7,7] by [-4,4]. Indicate the direction in which it is traced.

b.) Find the highest point on the curve. Justify your answer.

c.) Find all points of inflection on the curve. Justify your answer.

## Arc Length of a Parametrized Curve

**Let L be the length of a parametric curve that is traversed exactly once as t increases from $t_1$ to $t_2$,**

**If dx/dt and dy/dt are continuous functions of t, then**

$$L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

**Find the length of the astroid**

$$x = \cos^3 t,\ y = \sin^3 t,\ 0 \le t \le 2\pi$$

Given $x = -\sqrt{t+1}$ and $y = \sqrt{3t}$

a.) Find dy/dx

b.) Is the curve increasing or decreasing?

c.) Find the slope at t = 3.

d.) Write the equation of the tangent line at t = 3.

e.) Does the curve have any horizontal tangents?

f.) Does this curve have any vertical tangents?

g.) Find $d^2y/dx^2$.

h.) Concave up or concave down?

i.) Find the arc length from 0 to 4.

Given $x = 2 - t$ and $y = t^3 - 4t$

a.) Find $dy/dx$

b.) Find maximum or minimum values of the curve.

c.) Find $d^2y/dx^2$

d.) Concave up or concave down? Where?

e.) Points of inflection?

f.) Find the curve length from $t = 0$ $t = 2$.

Suppose $\frac{dy}{dx} = \frac{3t + 1}{t^2 - 4}$

Find the slope at t = 2.